

On the Significance of the Electroweak Precision Data

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Abstract

We elaborate on a recently suggested effective Lagrangian for charged-current and neutral-current electroweak interactions which in comparison with the standard electroweak theory contains three free parameters $\Delta x, \Delta y, \varepsilon$ which quantify different sources for violations of $SU(2)$ symmetry. Within the standard $SU(2)_I \times U(1)_Y$ electroweak theory, we present both exact and very much refined approximate analytical one-loop expressions for these parameters in terms of the canonical input, $G_\mu, M_Z, \alpha(M_Z^2)$, the top-quark mass, m_t , and the Higgs-boson mass, M_H . We reemphase the importance of discriminating between the *empirically well-known purely fermionic* (vacuum polarization) contributions to $\Delta x, \Delta y, \varepsilon$ and the *empirically unknown bosonic* ones with respect to present and future electroweak precision tests. The parameters Δx and ε are hardly affected by standard bosonic corrections, while the full one-loop results for Δy differ appreciably from the ones obtained by taking into account fermion loops only. A detailed comparison with the experimental data on $M_{W^\pm}/M_Z, \bar{s}_W^2, \Gamma_l$ shows that these data start to become accurate enough to be sensitive to standard (bosonic) contributions to Δy beyond fermion loops.

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1 Introduction

We have recently given [1] an effective Lagrangian for neutral- and charged-current electroweak vector-boson fermion interactions which contains three additional free parameters in comparison with the standard electroweak $SU(2)_I \times U(1)_Y$ gauge theory. The three parameters, called $\Delta x, \Delta y, \varepsilon$, specify the strengths of various $SU(2)$ -violating terms in the underlying Lagrangian. They are related to the charged and neutral boson masses, the charged-current and neutral-current couplings and a kinetic mixing term in the neutral-current sector, respectively. The purpose for analysing this effective Lagrangian is (at least) twofold [1]:

- (i) The effective Lagrangian, when evaluated at tree level, leads to predictions for the observables¹ $M_{W\pm}/M_Z$, \bar{s}_W^2 , Γ_l which differ from the standard ones due to the presence of the additional parameters $\Delta x, \Delta y$, and ε . Comparison with experiment leads to constraints in $\Delta x, \Delta y, \varepsilon$, which allow one to quantify to what extent the fundamental $SU(2)_I \times U(1)_Y$ symmetry of the standard electroweak theory in the vector-boson fermion sector is actually valid in nature.
- (ii) Within the standard electroweak theory the parameters $\Delta x, \Delta y, \varepsilon$ can be used to specify the one-loop radiative corrections in terms of the empirically known and unknown input parameters G_μ , $\alpha(M_Z^2)$, M_Z and m_t , M_H , respectively. The parameters $\Delta x, \Delta y, \varepsilon$ are well suited to implement the 1988 suggestion [2] of Gounaris and one of the present authors of clearly discriminating the one-loop effects originating from the *empirically well-known* interactions of the vector bosons with (light) fermions and the *empirically unknown* interactions of the bosons among each other. Evaluating the loop corrections to $\Delta x, \Delta y, \varepsilon$ by taking into account fermion loops only and comparing with the full one-loop results allows one to set the scale [2] for the accuracy which has to be reached in precision experiments if the standard theory is to be quantitatively tested beyond fermion loops. In other words, by comparing both the fermion-loop predictions as well as the full one-loop predictions with the electroweak precision data for $\Delta x, \Delta y, \varepsilon$ (obtained from $M_{W\pm}/M_Z$, \bar{s}_W^2 , and the leptonic width, Γ_l) allows one to judge in how far precision data are able to *isolate* and thus to *directly measure* standard (bosonic) contributions beyond the empirically well-known fermion loops. More generally, comparison of the dominant-fermion-loop predictions with the data quantifies the extent to which additional effects, standard or non-standard ones, are required by precision data.

The present paper expands on previous work [1, 3]. We explicitly give the complete analytic one-loop expressions for $\Delta x, \Delta y, \varepsilon$ in the standard $SU(2)_I \times U(1)_Y$ electroweak theory and explicitly compare the fermion-loop contributions with the full one-loop results. It turns out that the parameters $\Delta x, \Delta y, \varepsilon$ are particularly well-suited parameters for

¹We use the standard notation, $M_{W\pm}$ and M_Z , for the masses of charged and neutral weak bosons, respectively, \bar{s}_W^2 for the square of the weak leptonic mixing angle measured at $q^2 = M_Z^2$ and Γ_l for the leptonic decay width of the Z boson, etc. In the present paper we confine our analysis to the mentioned observables, as these are particularly simple ones which do not involve important hadronic (gluonic) effects.

identifying contributions beyond fermion loops. While ε and Δx are so strongly dominated by fermion-loop contributions that experiments will hardly ever be able to discriminate fermion loops from standard full one-loop results, the standard bosonic contributions to Δy , in contrast, turn out to be quite large. In fact, with the present data we are starting to be able to discriminate Δy as calculated from fermion loops alone from Δy as obtained by taking into account bosonic self-couplings as well. In this respect, it is of interest that Δy is the only one of the three parameters which contains the process specific vertex and box corrections to muon decay which enter the analysis of radiative effects via the input parameter G_μ . Accordingly, we conclude that the neutral-current precision data at the Z resonance start to quantitatively test this important charged-current one-loop correction to muon decay.

In Sect. 2, we briefly rederive our effective Lagrangian. In Sect. 3, we present exact and approximate explicit one-loop formulae for Δx , Δy , ε and M_{W^\pm}/M_Z , \bar{s}_W^2 , Γ_l within the standard electroweak theory. Our approximate expressions for Δx , Δy , ε , which are based on asymptotic expansions for $m_t, M_H \rightarrow \infty$, have a transparent and compact analytic form and are sufficiently accurate for all practical purposes. The analytic expressions for the remainder terms are collected in App. A, while App. B lists a few simple auxiliary functions. In Sect. 4, we compare the most recent empirical results for M_{W^\pm}/M_Z , \bar{s}_W^2 , Γ_l with the full one-loop and the fermion-loop predictions. Final conclusions are drawn in Sect. 5.

2 The effective Lagrangian

Introducing a charged W^\pm vector-boson of mass M_W and coupling g_{W^\pm} to the weak isospin current, j_μ^\pm , we have for charged-current interactions

$$\mathcal{L}_C = -\frac{1}{2}W^{+\mu\nu}W_{\mu\nu}^- + \frac{g_{W^\pm}}{\sqrt{2}}(j_\mu^+W^{+\mu} + h.c.) + M_{W^\pm}^2W_\mu^+W^{-\mu}. \quad (1)$$

In the transition to the neutral-current sector, we allow for breaking of $SU(2)$ symmetry in the coupling via the parameter y ,

$$g_{W^\pm}^2 = yg_{W^0}^2 = (1 + \Delta y)g_{W^0}^2, \quad (2)$$

and in the mass via the parameter x ,

$$M_{W^\pm}^2 = xM_{W^0}^2 = (1 + \Delta x)M_{W^0}^2. \quad (3)$$

In addition, we introduce current mixing of strength λ with the photon field² à la Hung-Sakurai [4]

$$\mathcal{L}_{mix} = \lambda A_{\mu\nu}W^{3\mu\nu}. \quad (4)$$

Upon diagonalisation, the neutral-current Lagrangian in the physical base reads

$$\begin{aligned} \mathcal{L}_N = & -\frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} + \frac{1}{2}\frac{M_{W^0}^2}{1-\lambda^2}Z_\mu Z^\mu - \frac{1}{4}A_{\mu\nu}A^{\mu\nu} \\ & -ej_{em}^\mu A_\mu - \frac{g_{W^0}}{\sqrt{1-\lambda^2}}\left(j^3 - \lambda\frac{e}{g_{W^0}}j_{em}\right)^\mu Z_\mu. \end{aligned} \quad (5)$$

²Equivalently, one may work in the W^3B base [3].

The electromagnetic coupling at the Z mass has been denoted by e with

$$\frac{e^2(M_Z^2)}{4\pi} = \alpha(M_Z^2) \approx 1/129, \quad (6)$$

where $\alpha(M_Z^2)$ includes the ‘running’ of electromagnetic vacuum polarization induced by the light fermions [5]. Upon introducing the empirical weak angle at the Z mass, \bar{s}_W^2 , via

$$\bar{s}_W^2 \equiv \lambda \frac{e}{g_{W^0}}, \quad (7)$$

and replacing λ by ε via

$$\lambda \equiv \frac{e}{g_{W^0}}(1 - \varepsilon), \quad (8)$$

we have

$$\bar{s}_W^2 = \frac{e^2}{g_{W^0}^2}(1 - \varepsilon). \quad (9)$$

The neutral-current interaction Lagrangian (5) then becomes [1]

$$\begin{aligned} \mathcal{L}_N = & -\frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} + \frac{1}{2} \frac{M_{W^0}^2}{1 - \bar{s}_W^2(1 - \varepsilon)} Z_\mu Z^\mu - \frac{1}{4}A_{\mu\nu}A^{\mu\nu} \\ & - e j_{em}^\mu A_\mu - \frac{g_{W^0}}{\sqrt{1 - \bar{s}_W^2(1 - \varepsilon)}} (j_3^\mu - \bar{s}_W^2 j_{em}^\mu) Z_\mu. \end{aligned} \quad (10)$$

In total, we have introduced two parameters in the charged-current sector, M_{W^\pm} , g_{W^\pm} , and four parameters related to the neutral-current sector, M_{W^0} (or, equivalently Δx in (3)), g_{W^0} (or, equivalently Δy in (2)), $e(M_Z^2)$, and λ (or equivalently ε in (8)). The weak mixing angle \bar{s}_W^2 is related to these parameters according to (9).

The standard electroweak interaction Lagrangian [6] for vector-boson fermion interactions is contained in (10) in the limit of $\varepsilon, \Delta x, \Delta y \rightarrow 0$. This is also evident from the expressions for the observable parameters M_{W^\pm}/M_Z , \bar{s}_W^2 , and the leptonic Z width, Γ_l , in terms of the canonical input, the Fermi coupling measured in muon decay

$$\frac{8G_\mu}{\sqrt{2}} = \frac{g_{W^\pm}^2}{M_{W^\pm}^2}, \quad (11)$$

the Z mass, M_Z , and the electromagnetic coupling, $\alpha(M_Z^2)$, at the Z resonance.

From the Lagrangian one easily obtains

$$\begin{aligned} \bar{s}_W^2 (1 - \bar{s}_W^2) &= \frac{\pi\alpha(M_Z^2)}{\sqrt{2}G_\mu M_Z^2} \frac{y}{x} (1 - \varepsilon) \frac{1}{\left(1 + \frac{\bar{s}_W^2}{1 - \bar{s}_W^2} \varepsilon\right)}, \\ \frac{M_{W^\pm}^2}{M_Z^2} &= (1 - \bar{s}_W^2) x \left(1 + \frac{\bar{s}_W^2}{1 - \bar{s}_W^2} \varepsilon\right), \\ \Gamma_l &= \frac{G_\mu M_Z^3}{24\pi\sqrt{2}} \left(1 + (1 - 4\bar{s}_W^2)^2\right) \frac{x}{y} \left(1 - \frac{3\alpha}{4\pi}\right). \end{aligned} \quad (12)$$

A QED correction factor is included in Γ_l in agreement with the definition of Γ_l used in the analysis of experimental data. Omitting this factor and taking the limit $\Delta x, \Delta y, \varepsilon \rightarrow 0$ indeed yields the well-known standard tree-level relations.

A comparison of (12) with experiment clearly allows to constrain $\Delta x, \Delta y, \varepsilon$, and accordingly allows one to quantify the extent to which the standard $SU(2)_I \times U(1)_Y$ symmetry is supported by empirical data.

Alternatively, the Lagrangians (1) and (10) may be interpreted as one-loop effective Lagrangians of the standard electroweak theory. As previously noted, the standard radiative corrections to M_{W^\pm}/M_Z , \bar{s}_W^2 , Γ_l may indeed be incorporated by appropriate specification of the $SU(2)$ -breaking parameters, $\Delta x, \Delta y, \varepsilon$. The explicit expressions and very accurate approximations for $\Delta x, \Delta y, \varepsilon$ in terms of the known input G_μ , $\alpha(M_Z^2)$ and M_Z and the unknown parameters m_t and M_H are given in the Sect. 3.

We close this section by noting that our parameters $\Delta x, \Delta y, \varepsilon$, introduced and defined in terms of $SU(2)$ -symmetry properties of electroweak interactions are simple linear combinations of the parameters ε_{Ni} ($i = 1, 2, 3$) introduced in Ref. [7] by the requirement of isolating the quadratic m_t -dependence in the one-loop standard formulae for these parameters,

$$\Delta x = \varepsilon_{N1} - \varepsilon_{N2}, \quad \Delta y = -\varepsilon_{N2}, \quad \varepsilon = -\varepsilon_{N3}. \quad (13)$$

These relations between the ε_{Ni} and our parameters establish the meaning of the ε_{Ni} with respect to $SU(2)$ -symmetry properties of electroweak interactions.

3 Analytic results for the parameters $\Delta x, \Delta y, \varepsilon$

In this section, we present the results for $\Delta x, \Delta y, \varepsilon$ within the electroweak standard model which are obtained by analyzing the decays $\mu^- \rightarrow \nu_\mu e^- \bar{\nu}_e$ and $Z \rightarrow l\bar{l}$ ($l = e, \mu, \tau$). More precisely, one expands (12) in linear order in $\Delta x, \Delta y, \varepsilon$ to obtain [1]

$$\begin{aligned} \bar{s}_W^2 &= s_0^2 \left[1 - \frac{1}{c_0^2 - s_0^2} \varepsilon - \frac{c_0^2}{c_0^2 - s_0^2} (\Delta x - \Delta y) \right], \\ \frac{M_{W^\pm}}{M_Z} &= c_0 \left[1 + \frac{s_0^2}{c_0^2 - s_0^2} \varepsilon + \frac{c_0^2}{2(c_0^2 - s_0^2)} \Delta x - \frac{s_0^2}{2(c_0^2 - s_0^2)} \Delta y \right], \\ \Gamma_l &= \Gamma_l^{(0)} \left[1 + \frac{8s_0^2(1 - 4s_0^2)}{(c_0^2 - s_0^2)(1 + (1 - 4s_0^2)^2)} \varepsilon + \frac{2(c_0^2 - s_0^2 - 4s_0^4)}{(c_0^2 - s_0^2)(1 + (1 - 4s_0^2)^2)} (\Delta x - \Delta y) \right], \end{aligned} \quad (14)$$

where

$$\begin{aligned} s_0^2(1 - s_0^2) &\equiv c_0^2 s_0^2 = \frac{\pi \alpha(M_Z^2)}{\sqrt{2} G_\mu M_Z^2}, \\ \Gamma_l^{(0)} &= \frac{\alpha(M_Z^2) M_Z}{48 s_0^2 c_0^2} \left[1 + (1 - 4s_0^2)^2 \right] \left(1 + \frac{3\alpha}{4\pi} \right), \end{aligned} \quad (15)$$

and identifies $\Delta x, \Delta y, \varepsilon$ by a comparison with the electroweak radiative corrections to M_{W^\pm}/M_Z , \bar{s}_W^2 , Γ_l within the standard model. For the explicit calculations of the corresponding one-loop radiative corrections we have applied well-known standard techniques, which are e.g. summarized in Ref. [8].

The contributions to Δx , Δy , ε are decomposed into gauge invariant subsets as far as possible. As already emphasized, we separate those contributions from the bulk of the radiative corrections which originate from Feynman graphs containing a closed fermion loop. Obviously this (gauge invariant) part is universal – as it enters merely via vacuum polarization – and is denoted by $a_{\text{ferm}}^{\text{univ}}$ ($a = \Delta x, \Delta y, \varepsilon$). The remaining part of the radiative corrections is split into a bosonic universal contribution, $a_{\text{bos}}^{\text{univ}}$, and two process dependent ones, a^{WPD} and a^{ZPD} , which are related to the muon (WPD) and the Z (ZPD) decay, respectively. Accordingly, we have the decomposition

$$a = a_{\text{ferm}}^{\text{univ}} + a_{\text{bos}}^{\text{univ}} + a^{\text{WPD}} + a^{\text{ZPD}}, \quad \text{with } a = \Delta x, \Delta y, \varepsilon. \quad (16)$$

Note that the (gauge invariant) decomposition of bosonic radiative corrections into process dependent and independent parts is not uniquely determined by the criterion of gauge invariance so that supplementary conditions have to be imposed. Various suggestions and methods on this subject were accordingly given in the literature by different authors [9, 10, 11]. Here we decided to apply the so-called ‘pinch technique’ [11] which provides a technically particularly simple prescription for the decomposition of transition amplitudes into gauge invariant self-energy, vertex, and box contributions.

We split the universal parts, $a_{\text{ferm/bos}}^{\text{univ}}$, further into ‘dominant’ parts, $a_{\text{ferm/bos}}^{\text{univ}}(\text{dom})$, and remainders, $a_{\text{ferm/bos}}^{\text{univ}}(\text{rem})$,

$$a_{\text{ferm/bos}}^{\text{univ}} = a_{\text{ferm/bos}}^{\text{univ}}(\text{dom}) + a_{\text{ferm/bos}}^{\text{univ}}(\text{rem}), \quad (17)$$

where $a_{\text{ferm/bos}}^{\text{univ}}(\text{dom})$ is defined as the asymptotic expansion of the full universal contribution, $a_{\text{ferm/bos}}^{\text{univ}}$, in the limit of a heavy top quark and a heavy Higgs boson, i.e. for $t \rightarrow \infty$, $h \rightarrow \infty$ with

$$t = \frac{m_t^2}{M_Z^2}, \quad h = \frac{M_H^2}{M_Z^2}, \quad (18)$$

keeping terms up to constant order. As will be discussed below these asymptotic formulae approximate $a_{\text{ferm/bos}}^{\text{univ}}$ very well in the relevant regions for t and h , so that the terminology ‘dominant’ is justified. In this context we mention that the remaining (light) fermions, i.e. all leptons and the light quarks, are taken exactly massless since $\alpha(M_Z^2)$ already includes all sizeable effects arising from light fermion masses.

The explicit expressions for the fermionic contributions read

$$\begin{aligned} \Delta x_{\text{ferm}}^{\text{univ}}(\text{dom}) &= \frac{\alpha(M_Z^2)}{8\pi s_0^2} \left[\frac{3}{2c_0^2} t + \log(t) + 6 \log(c_0^2) - \frac{197}{18} + \frac{103}{9c_0^2} - \frac{40s_0^2}{9} - \frac{160s_0^4}{9} \right], \\ \Delta y_{\text{ferm}}^{\text{univ}}(\text{dom}) &= \frac{\alpha(M_Z^2)}{8\pi s_0^2} \left[\log(t) + 6 \log(c_0^2) - \frac{13}{2} + \frac{40s_0^2}{3} - \frac{160s_0^4}{9} \right], \\ \varepsilon_{\text{ferm}}^{\text{univ}}(\text{dom}) &= \frac{\alpha(M_Z^2)}{24\pi s_0^2} \left[\log(t) - \frac{125}{6} + 40s_0^2 - \frac{160s_0^4}{3} \right], \end{aligned} \quad (19)$$

which of course include the well-known m_t^2 [12] and $\log m_t$ terms already given in Ref. [1]. An analogous statement holds for the $\log M_H$ terms of the universal bosonic contributions

$$\Delta x_{\text{bos}}^{\text{univ}}(\text{dom}) = \frac{\alpha(M_Z^2)}{16\pi s_0^2} \left[-\frac{3s_0^2}{c_0^2} \log(h) - \frac{s_0^2}{c_0^4} \left(\frac{233}{6} - \frac{1591s_0^2}{6} + 500s_0^4 - \frac{1060s_0^6}{3} + 80s_0^8 \right) \right]$$

$$\begin{aligned}
& + \left(\frac{23}{2} - \frac{68s_0^2}{3} + 8s_0^4 + 3s_0^6 \right) \frac{\log(c_0^2)}{c_0^6} - (3 - 4s_0^2) \\
& \times (33 - 44s_0^2 + 12s_0^4) \frac{f_2(c_0^2)}{6c_0^6} - (33 + 22s_0^2 - 100s_0^4 + 40s_0^6) f_1(c_0^2) \Big], \\
\Delta y_{\text{bos}}^{\text{univ}}(dom) &= \frac{\alpha(M_Z^2)}{16\pi s_0^2} \left[\frac{1}{c_0^4} \left(40 - \frac{1363s_0^2}{6} + \frac{1139s_0^4}{2} - 704s_0^6 + \frac{1204s_0^8}{3} - 80s_0^{10} \right) \right. \\
& - \left(1 - \frac{19s_0^2}{2} - \frac{32s_0^4}{3} + 6s_0^6 - 8s_0^8 \right) \frac{\log(c_0^2)}{s_0^2 c_0^6} - (3 - 4s_0^2) \\
& \times (33 - 44s_0^2 + 12s_0^4) \frac{f_2(c_0^2)}{6c_0^6} - (7 + 78s_0^2 - 124s_0^4 + 40s_0^6) f_1(c_0^2) \Big], \\
\varepsilon_{\text{bos}}^{\text{univ}}(dom) &= \frac{\alpha(M_Z^2)}{48\pi s_0^2} \left[-\log\left(\frac{h}{c_0^2}\right) + \frac{1063}{6} - 760s_0^2 + 820s_0^4 - 240s_0^6 \right. \\
& \left. + (105 - 406s_0^2 + 420s_0^4 - 120s_0^6) f_1(c_0^2) \right]. \tag{20}
\end{aligned}$$

The formulae for $a_{\text{ferm}/\text{bos}}^{\text{univ}}(rem)$ and the auxiliary functions $f_1(x)$, $f_2(x)$ appearing in (20) are listed in App. A and B, respectively. According to (20), the dominant universal bosonic contributions $\Delta x_{\text{bos}}^{\text{univ}}(dom)$ and $\varepsilon_{\text{bos}}^{\text{univ}}(dom)$ depend on the Higgs mass via $\log(h)$, while $\Delta y_{\text{bos}}^{\text{univ}}(dom)$ is a constant in the sense of being independent of h .

The process dependent parts for the Z decay are obviously independent of the top-quark and Higgs-boson masses and are given by

$$\begin{aligned}
\Delta x^{\text{ZPD}} &= -\frac{\alpha(M_Z^2)s_0^4}{4\pi c_0^2} [11 + 16C_1], \\
\varepsilon^{\text{ZPD}} &= \frac{\alpha(M_Z^2)}{4\pi s_0^2} \left[(1 - 6s_0^2 + 16s_0^4 - 8s_0^6) \frac{2C_1}{c_0^2} - (2 - s_0^2)^2 C_2 - 2c_0^4(3 - s_0^2)C_3 \right. \\
& \quad \left. - \left(\frac{5}{2} - s_0^2 \right) \left(\log(c_0^2) - 2c_0^2 f_1(c_0^2) \right) + \frac{17}{8c_0^2} - \frac{27s_0^2}{2c_0^2} + \frac{57s_0^4}{2c_0^2} - \frac{13s_0^6}{c_0^2} \right], \\
\Delta y^{\text{ZPD}} &= \frac{c_0^2}{s_0^2} \Delta x^{\text{ZPD}} + 2\varepsilon^{\text{ZPD}}. \tag{21}
\end{aligned}$$

The explicit formulae for the constants C_1 , C_2 , C_3 can be found in App. B. As already mentioned, only Δy gets process dependent contributions induced by vertex and box corrections to muon decay,

$$\Delta y^{\text{WPD}} = \frac{\alpha(M_Z^2)}{8\pi s_0^4} [4s_0^4 + (4s_0^4 - 1) \log(c_0^2)], \tag{22}$$

so that Δx^{WPD} and ε^{WPD} are identically zero,

$$\Delta x^{\text{WPD}} = \varepsilon^{\text{WPD}} = 0. \tag{23}$$

Recall that all genuine vertex corrections to the Z-boson and the muon decay are contained in a^{ZPD} and a^{WPD} , respectively, which is for example reflected by the appearance of the

| m_t/GeV | $\Delta x_{\text{ferm}}^{\text{univ}}$ | $\Delta x_{\text{ferm}}^{\text{univ}}(dom)$ | M_H/GeV | $\Delta x_{\text{bos}}^{\text{univ}}$ | $\Delta x_{\text{bos}}^{\text{univ}}(dom)$ |
|------------------|---|--|------------------|--|---|
| 100 | 3.21×10^{-3} | 3.90×10^{-3} | 100 | 2.17×10^{-3} | 2.93×10^{-3} |
| 140 | 7.50×10^{-3} | 7.80×10^{-3} | 300 | 1.39×10^{-3} | 1.61×10^{-3} |
| 220 | 17.92×10^{-3} | 18.04×10^{-3} | 1000 | 0.12×10^{-3} | 0.16×10^{-3} |
| m_t/GeV | $\Delta y_{\text{ferm}}^{\text{univ}}$ | $\Delta y_{\text{ferm}}^{\text{univ}}(dom)$ | M_H/GeV | $\Delta y_{\text{bos}}^{\text{univ}}$ | $\Delta y_{\text{bos}}^{\text{univ}}(dom)$ |
| 100 | -8.27×10^{-3} | -7.69×10^{-3} | 100 | -0.26×10^{-3} | -0.37×10^{-3} |
| 140 | -7.05×10^{-3} | -6.80×10^{-3} | 300 | -0.35×10^{-3} | -0.37×10^{-3} |
| 220 | -5.68×10^{-3} | -5.59×10^{-3} | 1000 | -0.37×10^{-3} | -0.37×10^{-3} |
| m_t/GeV | $\varepsilon_{\text{ferm}}^{\text{univ}}$ | $\varepsilon_{\text{ferm}}^{\text{univ}}(dom)$ | M_H/GeV | $\varepsilon_{\text{bos}}^{\text{univ}}$ | $\varepsilon_{\text{bos}}^{\text{univ}}(dom)$ |
| 100 | -6.25×10^{-3} | -6.35×10^{-3} | 100 | -2.80×10^{-3} | -3.17×10^{-3} |
| 140 | -6.00×10^{-3} | -6.05×10^{-3} | 300 | -3.59×10^{-3} | -3.66×10^{-3} |
| 220 | -5.63×10^{-3} | -5.64×10^{-3} | 1000 | -4.19×10^{-3} | -4.20×10^{-3} |

Table 1: Comparison of the approximations, $a_{\text{ferm/bos}}^{\text{univ}}(dom)$, for the universal parts of Δx , Δy , ε with the exact values. The latter are obtained according to (17) by evaluating the remainder terms, $a_{\text{ferm/bos}}^{\text{univ}}(rem)$, given in App. A.

corresponding three-point functions C_1, C_2, C_3 in a^{ZPD} (see App. B). Of course, if we had used finite external lepton-masses a^{ZPD} and a^{WPD} would have also contained the complete dependence on these masses.

By inspection of Tab. 1 we get an impression of the quality of the approximations $a_{\text{ferm/bos}}^{\text{univ}} \approx a_{\text{ferm/bos}}^{\text{univ}}(dom)$. It turns out that the differences between $a_{\text{ferm/bos}}^{\text{univ}}(dom)$ and $a_{\text{ferm/bos}}^{\text{univ}}$ are small compared with (present and future) experimental errors, even for relatively low values of m_t and M_H . More specifically, the differences are $\lesssim 0.5 \times 10^{-3}$, except for very low values of $m_t, M_H \sim 100 \text{ GeV}$. Recall that the approximations are constructed such that they approach the exact values asymptotically for $m_t, M_H \gg M_Z$.

In order to explicitly display the magnitude of the different contributions to Δx , Δy , ε , we also list the corresponding numerical expressions (based on $c_0 = 0.8768$):

$$\begin{aligned}
\Delta x_{\text{ferm}}^{\text{univ}}(dom) &= (+0.52 + 1.34 \log(t) + 2.61 t) \times 10^{-3}, \\
\Delta y_{\text{ferm}}^{\text{univ}}(dom) &= (-7.94 - 1.34 \log(t)) \times 10^{-3}, \\
\varepsilon_{\text{ferm}}^{\text{univ}}(dom) &= (-6.43 - 0.45 \log(t)) \times 10^{-3},
\end{aligned} \tag{24}$$

$$\begin{aligned}
\Delta x_{\text{bos}}^{\text{univ}}(dom) &= (+3.04 - 0.60 \log(h)) \times 10^{-3}, \\
\Delta y_{\text{bos}}^{\text{univ}}(dom) &= (-0.37) \times 10^{-3}, \\
\varepsilon_{\text{bos}}^{\text{univ}}(dom) &= (-3.13 - 0.22 \log(h)) \times 10^{-3},
\end{aligned} \tag{25}$$

$$\begin{aligned}
\Delta x^{\text{ZPD}} &= 0.09 \times 10^{-3}, \\
\Delta y^{\text{ZPD}} &= 8.43 \times 10^{-3}, \quad \Delta y^{\text{WPD}} = 5.46 \times 10^{-3}, \\
\varepsilon^{\text{ZPD}} &= 4.06 \times 10^{-3},
\end{aligned} \tag{26}$$

Note that Δx^{WPD} and ε^{WPD} vanish according to (23).

As mentioned, the process dependent parts of Δx , Δy , ε were extracted by means of the pinch technique [11]. Other procedures, previously used, lead to slightly different decompositions, e.g.

$$\begin{aligned}\Delta y^{\text{WPD}}|_{\text{Ref. [9]}} &\equiv \delta_{\text{vertex}} + \delta_{\text{box}} = \frac{\alpha(M_Z^2)}{4\pi s_0^2} \left[6 + \frac{2s_0^4 - 6s_0^2 + 7}{2s_0^2} \log(c_0^2) \right] \\ &= 7.34 \times 10^{-3},\end{aligned}\tag{27}$$

$$\begin{aligned}\Delta y^{\text{WPD}}|_{\text{Ref. [10]}} &\equiv \varepsilon_{\text{vertex}} + \varepsilon_{\text{box}} = \frac{\alpha(M_Z^2)}{4\pi s_0^2} \left[6s_0^2 - \frac{2s_0^4 - 10s_0^2 + 5}{2s_0^2} \log(c_0^2) \right] \\ &= 7.95 \times 10^{-3},\end{aligned}\tag{28}$$

of the (unique value of the) sum in (16).

It seems appropriate to add a brief remark on the basic parameters in our Lagrangians (1) and (10) at this point. The parameter Δx according to (3) describes global $SU(2)$ violation in the ratio of the charged, M_{W^\pm} , and (unmixed) neutral boson mass, M_{W^0} . Accordingly, Δx should be a process independent quantity in the $SU(2)_I \times U(1)_Y$ theory. This is indeed fulfilled, as according to (26) $\Delta x^{\text{ZPD}} \approx 0$. Similarly, process independence must hold for the mass ratio $M_{W^0}^2/M_Z^2$. From the expression for the Z-boson mass in (10), with (9) and (2), one obtains

$$\begin{aligned}z = \frac{M_{W^0}^2}{M_Z^2} &= 1 - \frac{e^2(M_Z^2)}{g_{W^\pm}^2(0)} (1 + \Delta y - 2\varepsilon) \\ &= 1 - \frac{e^2(M_Z^2)}{g_{W^\pm}^2(0)} \left(1 + \Delta y_{\text{ferm}}^{\text{univ}} + \Delta y_{\text{bos}}^{\text{univ}} - 2\varepsilon_{\text{ferm}}^{\text{univ}} - 2\varepsilon_{\text{bos}}^{\text{univ}} \right. \\ &\quad \left. + \Delta y^{\text{WPD}} + \frac{c_0^2}{s_0^2} \Delta x^{\text{ZPD}} \right),\end{aligned}\tag{29}$$

where the last formula in (21) was used in the second step. For clarity, we added the scale $g_{W^\pm}^2 \equiv g_{W^\pm}^2(0)$ as an argument, as $g_{W^\pm}^2$ in (1) according to (11) refers to the scale relevant for muon decay. Noting that according to the definition (2), Δy^{WPD} removes the muon-decay process-dependent part from $g_{W^\pm}^2(0)$, and neglecting the small contribution due to $\Delta x^{\text{ZPD}} \approx 0.1 \times 10^{-3}$, we indeed find that also the mass ratio z is a universal quantity at one-loop level in the standard theory. We note that (29) may be rewritten in terms of our canonical input parameter c_0 as

$$z = c_0^2 + \frac{s_0^2 c_0^2}{c_0^2 - s_0^2} (2\varepsilon + \Delta x - \Delta y).\tag{30}$$

A final comment in this context concerns the significance of the last relation in (21) itself. Upon neglecting Δx^{ZPD} , it reads $\varepsilon^{\text{ZPD}} \approx \Delta y^{\text{ZPD}}/2$. This is gratifying, as finally only two independent process-dependent (Z- and W-vertex) corrections remain, Δy^{ZPD} and Δy^{WPD} . The appearance of Δy^{ZPD} as a linear contribution to the mixing parameter ε , defined by (4) and (8), is understood as a consequence of the fact that mixing effects can be represented by vertex modifications (and vice versa) via linear field redefinitions [3].

We have compared our numerical results with the ones obtained by numerical evaluation based on the computer code ZFITTER [13] and on the formulae in Ref. [14]. We found good agreement in general. We note, however, a small discrepancy with the results obtained by evaluating the formulae in Ref. [14]. The discrepancy is due to the incorrect identification³ of $\alpha(M_Z^2)$ (which according to its empirical evaluation [5] contains the running of $\alpha(q^2)$ due to (light) fermion loops only) in Ref. [14] with the full expression for $\alpha(M_Z^2)$ which also contains (small) effects from the top-quark and the W bosons.

The fact that Δy is practically independent of M_H and that it is the only parameter which depends on the process specific corrections to muon decay (entering via the input parameter G_μ) underlines the usefulness of our set of parameters for the analysis of the experimental data. Moreover recall that the explicit formulae for Δx , Δy , ε given here, together with formula (14), represent the complete one-loop results for the observables M_{W^\pm}/M_Z , \bar{s}_W^2 , Γ_l within the electroweak standard model, which have to be compared with the implicitly given and relatively complex results existing in the literature.

We conclude our investigation of the parameters Δx , Δy , ε within the standard model with a short excursion to the related decay $Z \rightarrow \nu_l \bar{\nu}_l$ ($l = e, \mu, \tau$). Making use of the knowledge of Δx and Δy , the decay width for $Z \rightarrow \nu_l \bar{\nu}_l$, Γ_ν , including all $\mathcal{O}(\alpha)$ corrections is given by

$$\Gamma_\nu = \Gamma_\nu^{(0)} (1 + \Delta x - \Delta y + \delta_\nu^{rem}), \quad (31)$$

with

$$\begin{aligned} \Gamma_\nu^{(0)} &= \frac{\alpha(M_Z^2) M_Z}{24 s_0^2 c_0^2}, \\ \delta_\nu^{rem} &= \frac{\alpha(M_Z^2)}{8\pi c_0^2} \left[-48(1 - 2s_0^2)C_1 - 8c_0^2(2 - s_0^2)^2 C_2 \right. \\ &\quad \left. - 4c_0^2(5 - 2s_0^2) \log(c_0^2) - 55 + 96s_0^2 - 8s_0^4 \right], \end{aligned} \quad (32)$$

where δ_ν^{rem} accounts for an additional (process dependent) $SU(2)$ -symmetry breaking, which, for simplicity, was not incorporated into our effective Lagrangian. Note that all universal – both bosonic and fermionic – corrections to Γ_ν are included in $\Delta x - \Delta y$, whereas δ_ν^{rem} just corrects the ZPD parts of $\Delta x - \Delta y$ by

$$\delta_\nu^{rem} = 2.86 \times 10^{-3}. \quad (33)$$

4 Comparison with the experimental data

In our comparison between theory and experiment, as in Ref. [1], we will proceed in two steps. In a first step, we will directly compare the theoretical predictions (14) with the empirical data in the three-dimensional space of M_{W^\pm}/M_Z , \bar{s}_W^2 , Γ_l . In a second step, by using the inversion of (14), which is explicitly given in Ref. [1], we calculate the experimental values of the parameters Δx , Δy , ε from the data on M_{W^\pm}/M_Z , \bar{s}_W^2 , Γ_l and compare them with our theoretical predictions based on (16) etc.

³A correction of this point contained in Ref. [15] was brought to our attention upon circulating a preliminary version of the present paper.

The relevant experimental data from the four LEP collaborations and the CDF/UA2 value for the W mass are given by⁴

$$\begin{aligned} M_Z &= 91.1899 \pm 0.0044 \text{ GeV}, \\ M_{W^\pm}/M_Z &= 0.8814 \pm 0.0021, \\ \Gamma_l &= 83.98 \pm 0.18 \text{ MeV}, \\ g_V/g_A(\text{all asymmetries LEP}) &= 0.0711 \pm 0.0020. \end{aligned} \quad (34)$$

From the value of g_V/g_A we derive

$$\bar{s}_W^2(\text{all asymmetries LEP}) = 0.23223 \pm 0.00050. \quad (35)$$

Inclusion of the preliminary value derived from the measurement of the left-right asymmetry at the SLC by the SLD collaboration yields [17]

$$g_V/g_A(\text{all asymmetries LEP+SLD}) = 0.0737 \pm 0.0018, \quad (36)$$

and consequently

$$\bar{s}_W^2(\text{all asymmetries LEP+SLD}) = 0.23158 \pm 0.00045. \quad (37)$$

Both values (35) as well as (37) for \bar{s}_W^2 will be confronted with our theoretical predictions.

The high precision of the experimental data is best appreciated by comparing the data with the $\alpha(0)$ tree-level predictions based on M_Z and

$$\begin{aligned} \alpha(0) &= 1/137.0359895(61), \\ G_\mu &= 1.16639(2)10^{-5} \text{ GeV}^{-2}, \end{aligned} \quad (38)$$

as carried out e.g. in Ref. [1]. The figures shown there drastically demonstrate that the $\alpha(0)$ tree approximation is ruled out by several standard deviations.

Turning to a more refined analysis in Figs. 1-3, we show the three projections of the 68% C.L. volume defined by the data in the three-dimensional $(M_{W^\pm}/M_Z, \bar{s}_W^2, \Gamma_l)$ -space in comparison with various theoretical predictions. The point denoted by the symbol ‘star’ corresponds to the $\alpha(M_Z^2)$ -tree-level prediction obtained from (14) by putting $\Delta x, \Delta y, \varepsilon \rightarrow 0$ and using G_μ , M_Z , and [5]

$$\alpha(M_Z^2) = 1/128.87 \pm 0.12 \quad (39)$$

as input parameters. While the $\alpha(M_Z^2)$ -tree-level prediction only includes the vacuum-polarization effects of the light fermions (e^\pm, μ^\pm, τ^\pm and the five light quarks) in the photon propagator (the ‘running’ $\alpha(q^2)$ between $q^2 = 0$ and $q^2 = M_Z^2$), a complete treatment of fermion-loop corrections also affects the W^\pm and the Z propagators and has to include the top-quark loops. The corresponding predictions, obtained by inserting $a_{\text{ferm}}^{\text{univ}}$ ($a =$

⁴These data are the most recent preliminary data presented at the La Thuile and Moriond conferences, March 1994 [16, 17].

$\Delta x, \Delta y, \varepsilon$) from (19) and App. A into (14), are shown by the single lines⁵ in Figs. 1-3 for various values of m_t . Finally, by inserting the complete one-loop expressions for $\Delta x, \Delta y, \varepsilon$ from (19) to (23) and App. A into (14), one obtains the three connected lines in Figs. 1-3. They correspond to $M_H = 100, 300$ and 1000 GeV, respectively, while m_t varies from $m_t = 60$ GeV to 240 GeV. Note that all theoretical predictions are subject to the experimental uncertainty (39) of the input parameter $\alpha(M_Z^2)$. For the $\alpha(M_Z^2)$ -tree-level prediction this uncertainty is indicated by error bars in Figs. 1-3.

We draw the following conclusions from Figs. 1-3:

- (i) The data have reached sufficient precision to allow one to start the discrimination between the $\alpha(M_Z^2)$ -tree-level, the fermion-loop and the full one-loop prediction of the electroweak theory.
- (ii) The data start to establish a discrepancy from the fermion-loop predictions, i.e. they start to require contributions beyond the fermion-loop vacuum polarization to γ, Z and W^\pm propagation.
- (iii) Such additional contributions are provided by the standard ‘bosonic’ effects which contain vertex corrections depending on the trilinear boson self-couplings as well as vacuum-polarization effects depending on trilinear and quadrilinear boson self-couplings and the couplings of the Higgs scalar. In fact, the data show a clear tendency to agree with the standard predictions, provided a top quark exists in nature with a mass of the order of 150 GeV.

In Figs. 4-6, we show the experimental data for $\Delta x, \Delta y, \varepsilon$ derived from the data on $M_W/M_Z, \bar{s}_W^2, \Gamma_l$ by employing the inversion of (14) as given in Ref. [1]. As in Figs. 1-3, the data are compared with the fermion-loop predictions and the full one-loop standard results. Several interesting conclusions can immediately be drawn:

- (i) First of all, the $SU(2)$ violating parameters $\Delta x, \Delta y, \varepsilon$ in our basic Lagrangian (1), (10) are experimentally restricted to the magnitude of about 10×10^{-3} , which is the order of magnitude typically induced in these parameters by standard one-loop electroweak corrections.
- (ii) As specifically seen in Fig. 4, the fermion-loop predictions for ε and Δx hardly differ from the full one-loop results. In other words, the addition of the radiative effects originating from standard bosonic couplings of the vector bosons among each other and to the hypothetical Higgs scalar only leads to minor effects in ε and Δx . These cannot be resolved by present (and future) precision data.
- (iii) Figs. 5,6, however, reveal a spectacular difference between the fermion-loop and the full one-loop results. As inferred from in Figs. 5,6, the fermion-loop line of Fig. 4 lies far below the full one-loop M_H -dependent lines in three-dimensional $(\varepsilon, \Delta x, \Delta y)$ -space. The shift in Δy between the (uniquely determined) fermion-loop and the full one-loop results (by using the numerical formulae in (24) to (26)) is easily traced

⁵Note that the corresponding single lines in the figures of Ref. [1] were obtained by including only the top-quark contributions.

back to the sum of the process dependent corrections $\Delta y^{\text{WPD}} + \Delta y^{\text{ZPD}} \approx 14 \times 10^{-3}$, as the universal bosonic contribution, $\Delta y_{\text{bos}}^{\text{univ}}$, is negligibly small. The variable Δy thus appears to be the only one of our three parameters which is sensitive to standard bosonic effects. Moreover, these bosonic corrections are due to the sum of process specific vertex and box corrections to muon decay (entering via G_μ) and the process specific vertex corrections to Z decay into charged leptons. In other words, the neutral-current data at the Z resonance start to quantitatively isolate the standard radiative corrections to the charged-current muon-decay reaction in conjunction with the vertex corrections to Z decay.

In summary, the deviations between the data and the fermion-loop predictions in Figs. 1-3 find their origin in the fairly substantial W^\pm - and Z-vertex corrections quantified by Δy .

5 Conclusions

The empirical data on the M_{W^\pm} mass and the leptonic observables, \bar{s}_W^2 and Γ_l , at the Z mass, are analysed in terms of an effective Lagrangian allowing for $SU(2)$ breaking via the parameters Δx , Δy and ε . Systematically discriminating between (trivial) fermion-loop corrections to γ , Z and W^\pm propagation and full one-loop results we have given compact and simple analytic expressions for Δx , Δy , ε , and consequently for M_{W^\pm}/M_Z , \bar{s}_W^2 , Γ_l , in the standard $SU(2)_I \times U(1)_Y$ electroweak theory.

A comparison of the theoretical predictions with the most recent data from LEP, SLD and CDF/UA2 shows that $SU(2)$ breaking quantified by the parameters Δx , Δy , ε is restricted to the order of magnitude of $10 \cdot 10^{-3}$ typical for the order of magnitude of (standard) radiative one-loop corrections. This fact in itself constitutes a major triumph of the standard electroweak theory, as dramatic effects, previously speculated upon, such as effects owing to e.g. near-by compositeness, excited vector bosons etc., are excluded at a very high level of precision.

Moreover, the data have become sufficiently accurate to start to require genuine additional (virtual) effects beyond the $\alpha(M_Z^2)$ -tree-level and the full fermion-loop predictions. Such additional contributions are inherently contained in the standard theory. The difference between the fermion-loop and the full one-loop results is specifically traced back to important vertex corrections at the $W^\pm f \bar{f}'$ and the $Z f \bar{f}$ vertices. This is revealed by analysing the data in terms of the observable parameters Δx , Δy , ε derived from M_{W^\pm}/M_Z , \bar{s}_W^2 , Γ_l .

While present data are in excellent agreement with the standard theory and even start to test fine details of the virtual bosonic effects, the eventual direct verification of the boson self-couplings and the discovery of the Higgs scalar (or of whatever it stands for), apart from discovering the top quark, seem indispensable for completely revealing the structure of the electroweak phenomena in nature.

Appendix

A Remainders of the universal parts of Δx , Δy , ε

In Sect. 3 formula (16), we have defined the quantity $a_{\text{ferm}/\text{bos}}^{\text{univ}}(\text{rem})$ as the difference between the exact universal part $a_{\text{ferm}/\text{bos}}^{\text{univ}}$ and the corresponding asymptotic limits for a heavy top quark ($t \rightarrow \infty$) and a heavy Higgs boson ($h \rightarrow \infty$). Although the contributions of these ‘remainders’ can be neglected for most practical purposes (compare Tab. 1), for completeness we give their analytic expressions. For the fermionic contributions we obtain

$$\begin{aligned}
\Delta x_{\text{ferm}}^{\text{univ}}(\text{rem}) &= \frac{\alpha(M_Z^2)}{8\pi s_0^2 c_0^2} \left[-\frac{1}{2} + \frac{3s_0^2}{2} - \frac{232s_0^4}{27} + \frac{256s_0^6}{27} - \left(\frac{3}{2} + s_0^2 + \frac{272s_0^4}{9} - \frac{320s_0^6}{9} \right) t \right. \\
&\quad \left. \frac{t^2}{c_0^2} + (t - c_0^2)^2(t + 2c_0^2) \frac{1}{c_0^4} \log \left(1 - \frac{c_0^2}{t} \right) + \frac{f_1(t)}{1-4t} \left\{ c_0^2 \left(1 - \frac{32s_0^4}{9} \right) \right. \right. \\
&\quad \left. \left. - \left(5 - 2s_0^2 - \frac{64s_0^4 c_0^2}{9} \right) t + \left(4 + 2s_0^2 + \frac{544s_0^4}{9} - \frac{640s_0^6}{9} \right) t^2 \right\} \right], \\
\Delta y_{\text{ferm}}^{\text{univ}}(\text{rem}) &= \frac{\alpha(M_Z^2)}{8\pi s_0^2} \left[\frac{t^2}{c_0^4} + \left(\frac{3}{2} + 15s_0^2 - \frac{464s_0^4}{9} + \frac{320s_0^6}{9} \right) \frac{t}{c_0^2} - \frac{3}{2} + \frac{8s_0^2}{3} - \frac{256s_0^4}{27} \right. \\
&\quad \left. + \left(2 - \frac{3t}{c_0^2} + \frac{t^3}{c_0^6} \right) \log \left(1 - \frac{c_0^2}{t} \right) + \left\{ (1-2t) \left(1 - \frac{32s_0^4}{9} \right) \right. \right. \\
&\quad \left. \left. - 2t^2 \left(1 + 16s_0^2 - \frac{320s_0^4}{9} \right) \right\} \frac{f_1(t)}{1-4t} \right], \\
\varepsilon_{\text{ferm}}^{\text{univ}}(\text{rem}) &= \frac{\alpha(M_Z^2)}{8\pi s_0^2} \left[-\frac{19}{18} + \frac{232s_0^2}{27} - \frac{256s_0^4}{27} - (39 - 272s_0^2 + 320s_0^4) \frac{t}{9} + \{(1-2t) \right. \\
&\quad \left. \times (-3 + 32s_0^2 c_0^2) + 2t^2(39 - 272s_0^2 + 320s_0^4) \} \frac{f_1(t)}{9(1-4t)} \right], \quad (40)
\end{aligned}$$

and the bosonic contributions read

$$\begin{aligned}
\Delta x_{\text{bos}}^{\text{univ}}(\text{rem}) &= \frac{\alpha(M_Z^2)}{8\pi s_0^2 c_0^2} \left[\frac{47s_0^2}{12} - s_0^2 h + (1 - 2s_0^2) \frac{s_0^2 h^2}{6c_0^2} + \left(\frac{5c_0^2}{3} - \frac{3h}{2} + \frac{h^2}{2c_0^2} - \frac{h^3}{12c_0^4} \right) \log(c_0^2) \right. \\
&\quad \left. + s_0^2 \left(\frac{5}{2} - \frac{3h}{2} + (1 - 3s_0^2) \frac{h^2}{4c_0^2} + (3 - 2s_0^2) \frac{s_0^2 h^3}{12c_0^4} \right) \log(h) \right. \\
&\quad \left. + \left(4 + 3s_0^2 - (7 + 8s_0^2) \frac{h}{3} + (8 + 13s_0^2) \frac{h^2}{12} - (1 + 2s_0^2) \frac{h^3}{12} \right) \right. \\
&\quad \left. \times \frac{h}{h-4} f_2 \left(\frac{1}{h} \right) + \left(1 - \frac{h}{3c_0^2} + \frac{h^2}{12c_0^4} \right) h f_2 \left(\frac{c_0^2}{h} \right) \right], \\
\Delta y_{\text{bos}}^{\text{univ}}(\text{rem}) &= \frac{\alpha(M_Z^2)}{8\pi s_0^2} \left[-\frac{47}{12} + (7 - 4s_0^2) \frac{h}{4c_0^2} - \left(\frac{5}{6} - \frac{3h}{2} + \frac{3h^2}{4} - \frac{h^3}{6} \right) \log(c_0^2) \right. \\
&\quad \left. - (3 - 4s_0^2 + 2s_0^4) \frac{h^2}{6c_0^4} + \left(c_0^2 + (1 + 2c_0^6) \frac{h^4}{12c_0^6} - (11 - 3s_0^2) \frac{h}{2} \right) \right]
\end{aligned}$$

$$\begin{aligned}
& + (17 - 12s_0^2 + 3s_0^4) \frac{h^2}{4c_0^2} - (18 - 24s_0^2 + 15s_0^4 - 2s_0^6) \frac{h^3}{12c_0^4} \Bigg) \\
& \times \frac{1}{h - c_0^2} \log \left(\frac{h}{c_0^2} \right) - \left(3 - \frac{8h}{3} + \frac{13h^2}{12} - \frac{h^3}{6} \right) \frac{h}{h - 4} f_2 \left(\frac{1}{h} \right) \\
& + \left(1 - \frac{h}{3c_0^2} + \frac{h^2}{12c_0^4} \right) \frac{h}{c_0^2} f_2 \left(\frac{c_0^2}{h} \right) \Bigg], \\
\varepsilon_{\text{bos}}^{\text{univ}}(rem) &= \frac{\alpha(M_Z^2)}{8\pi s_0^2} \left[-\frac{67}{32} + h - \frac{h^2}{3} - \left(\frac{5}{6} - \frac{3h}{2} + \frac{3h^2}{4} - \frac{h^3}{6} \right) \log(h) \right. \\
& \left. - \left(3 - \frac{8h}{3} + \frac{13h^2}{12} - \frac{h^3}{6} \right) \frac{h}{h - 4} f_2 \left(\frac{1}{h} \right) \right]. \tag{41}
\end{aligned}$$

By definition, $a_{\text{ferm}/\text{bos}}^{\text{univ}}(rem)$ approach asymptotically zero with $t, h \rightarrow \infty$. In other words, all powers t^n, h^n ($n \geq 0$) cancel exactly in these asymptotic expansions.

B Auxiliary functions

Here we list the explicit expressions for the auxiliary functions which have been used in Sect. 3. The functions f_1 and f_2 are given by

$$\begin{aligned}
f_1(x) &= \text{Re} \left[\beta_x \log \left(\frac{\beta_x - 1}{\beta_x + 1} \right) \right], \quad \text{with } \beta_x = \sqrt{1 - 4x + i\epsilon}, \\
f_2(x) &= \text{Re} \left[\beta_x^* \log \left(\frac{1 - \beta_x^*}{1 + \beta_x^*} \right) \right],
\end{aligned}$$

more explicitly,

$$\begin{aligned}
0 < x < \frac{1}{4} : \quad f_1(x) &= f_2(x) = \sqrt{1 - 4x} \log \left(\frac{1 - \sqrt{1 - 4x}}{1 + \sqrt{1 - 4x}} \right), \\
x > \frac{1}{4} : \quad f_1(x) &= \sqrt{4x - 1} \left(2 \arctan \sqrt{4x - 1} - \pi \right), \\
f_2(x) &= 2\sqrt{4x - 1} \arctan \sqrt{4x - 1}. \tag{42}
\end{aligned}$$

The constants C_1, C_2, C_3 are shorthands for the scalar three-point integrals occuring in the process dependent parts of the Z decay,

$$\begin{aligned}
C_1 &= M_Z^2 \text{Re} \left[C_0(0, 0, M_Z^2, 0, M_Z, 0) \right] = -\frac{\pi^2}{12} = -0.8225, \\
C_2 &= M_Z^2 \text{Re} \left[C_0(0, 0, M_Z^2, 0, M_W, 0) \right] = \frac{\pi^2}{6} - \text{Re} \left[\text{Li}_2 \left(1 + \frac{1}{c_0^2} \right) \right] = -0.8037, \\
C_3 &= M_Z^2 \text{Re} \left[C_0(0, 0, M_Z^2, M_W, 0, M_W) \right] \\
&= \text{Re} \left[\log^2 \left(\frac{i\sqrt{4c_0^2 - 1} - 1}{i\sqrt{4c_0^2 - 1} + 1} \right) \right] = - \left(\pi - 2 \arctan \sqrt{4c_0^2 - 1} \right)^2 = -1.473. \tag{43}
\end{aligned}$$

The first three arguments of the C_0 function label the external momenta squared, the last three the inner masses of the corresponding vertex diagram.

References

- [1] M. Bilenky, K. Kolodziej, M. Kuroda, and D. Schildknecht, *Phys. Lett.* **B319** (1993) 319;
D. Schildknecht, *Proceedings of the International Conference on High Energy Physics*, Marseille, July 1993.
- [2] G. Gounaris and D. Schildknecht, *Z. Phys.* **C40** (1988) 447, *Z. Phys.* **C42** (1989) 107.
- [3] J.-L. Kneur, M. Kuroda, and D. Schildknecht, *Phys. Lett.* **B262** (1991) 93,
- [4] P.Q. Hung and J.J. Sakurai, *Nucl. Phys.* **B143** (1978) 81;
J.D. Bjorken, *Phys. Rev.* **D19** (1979) 335.
- [5] H. Burkhardt, F. Jegerlehner, G. Penso, and C. Verzegnassi, *Z. Phys.* **C43** (1989) 497.
- [6] S.L. Glashow, *Nucl. Phys.* **B22** (1961) 579;
S. Weinberg, *Phys. Rev. Lett.* **19** (1967) 1264;
A. Salam, in *Elementary Particle Theory*, ed. N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.
- [7] G. Altarelli, R. Barbieri, and F. Caravaglios, *Nucl. Phys.* **B405** (1993) 3;
G. Altarelli, CERN-TH 6867/93, *Schladming lecture*, February 1993.
- [8] A. Denner, *Fortschr. Phys.* **41** (1993) 307;
M. Böhm and A. Denner, *Radiative Corrections in the Electroweak Standard Model*, in *Proceedings of the Workshop on High Energy Phenomenology*, Mexico City, 1991, eds. R. Huerta and M.A. Perez (World Scientific, Singapore, 1992), p. 1.
- [9] D.C. Kennedy, *Nucl. Phys.* **B321** (1989) 83;
D.C. Kennedy and B.W. Lynn, *Nucl. Phys.* **B322** (1989) 1;
B.W. Lynn, Stanford University Report SU-ITP-867 (1989);
D.C. Kennedy, FNAL Report FERMI-CONF-91/271-T (1992).
- [10] M. Kuroda, G. Moulataka, and D. Schildknecht, *Nucl. Phys.* **B350** (1991) 25.
- [11] J.M. Cornwall, *Phys. Rev.* **D26** (1982) 1453;
J.M. Cornwall and J. Papavassiliou, *Phys. Rev.* **D40** (1989) 3474;
J. Papavassiliou, *Phys. Rev.* **D41** (1990) 3179;
G. Degrassi and A. Sirlin, *Phys. Rev.* **D46** (1992) 3104.
- [12] M. Veltman, *Nucl. Phys.* **B123** (1977) 89.
- [13] D. Bardin et al., CERN-TH. 6443/92.
- [14] V.A. Novikov, L.B. Okun, and M.I. Vysotsky, *Nucl. Phys.* **B397** (1993) 35.
- [15] V.A. Novikov, L.B. Okun, and M.I. Vysotsky, CERN-TH. 7071/93.

- [16] LEP collaborations, *La Thuile and Moriond Conferences*, March 1994.
- [17] SLD collaboration, *La Thuile and Moriond Conferences*, March 1994.

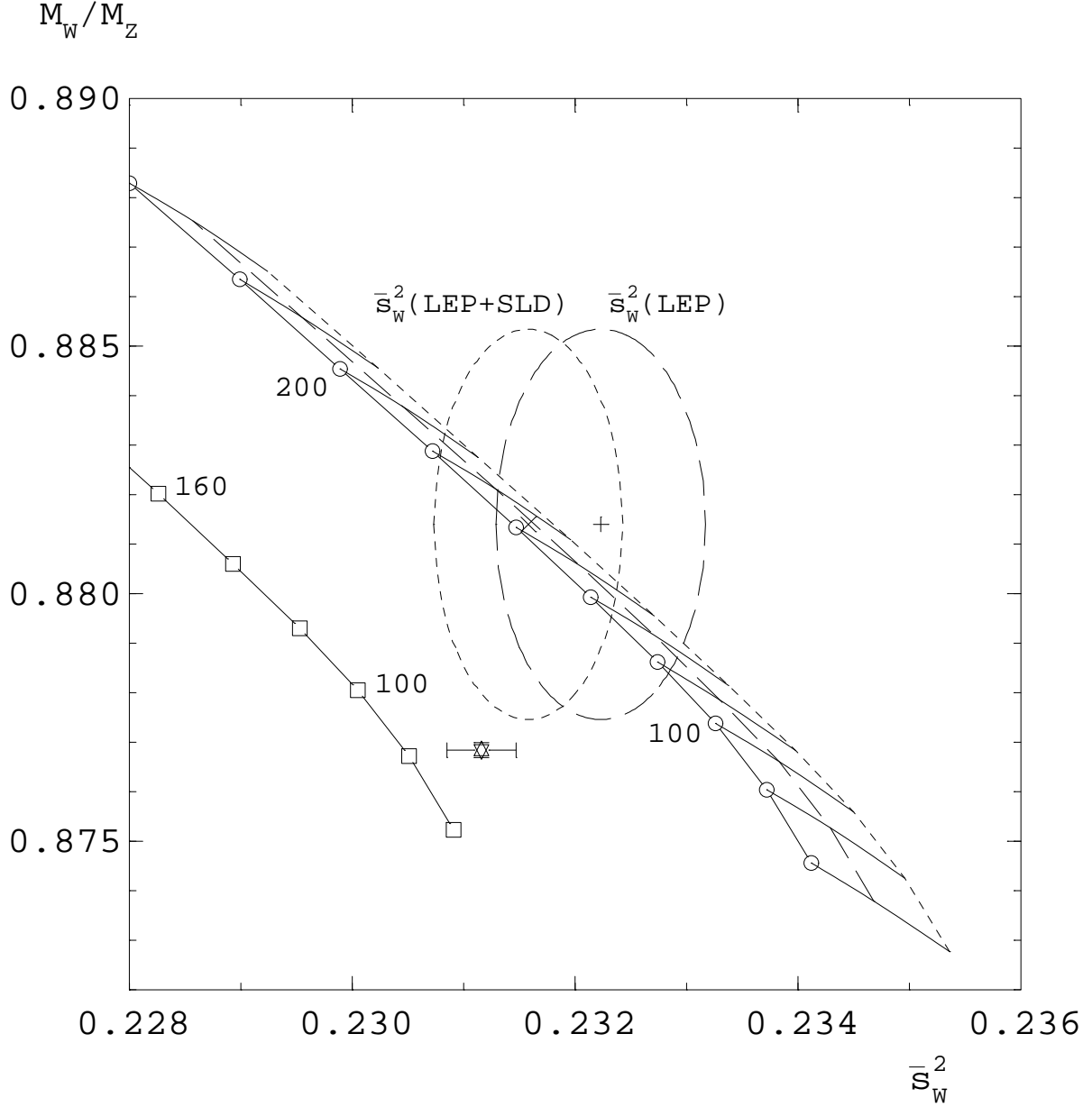


Figure 1: The experimental data are represented by the projection of the 68% C.L. volume in $(M_{W\pm}/M_Z, \bar{s}_W^2, \Gamma_l)$ -space into the $(M_{W\pm}/M_Z, \bar{s}_W^2)$ -plane. The symbol ‘star’ denotes the $\alpha(M_Z^2)$ -tree-level prediction. The fermion-loop prediction is shown by the single line, the squares indicating steps in 20 GeV in m_t . The full one-loop standard-model prediction is shown for Higgs-boson masses of $M_H = 100$ GeV (solid line), 300 GeV (long-dashed line), 1000 GeV (short-dashed line), the circles indicating steps in 20 GeV in m_t .

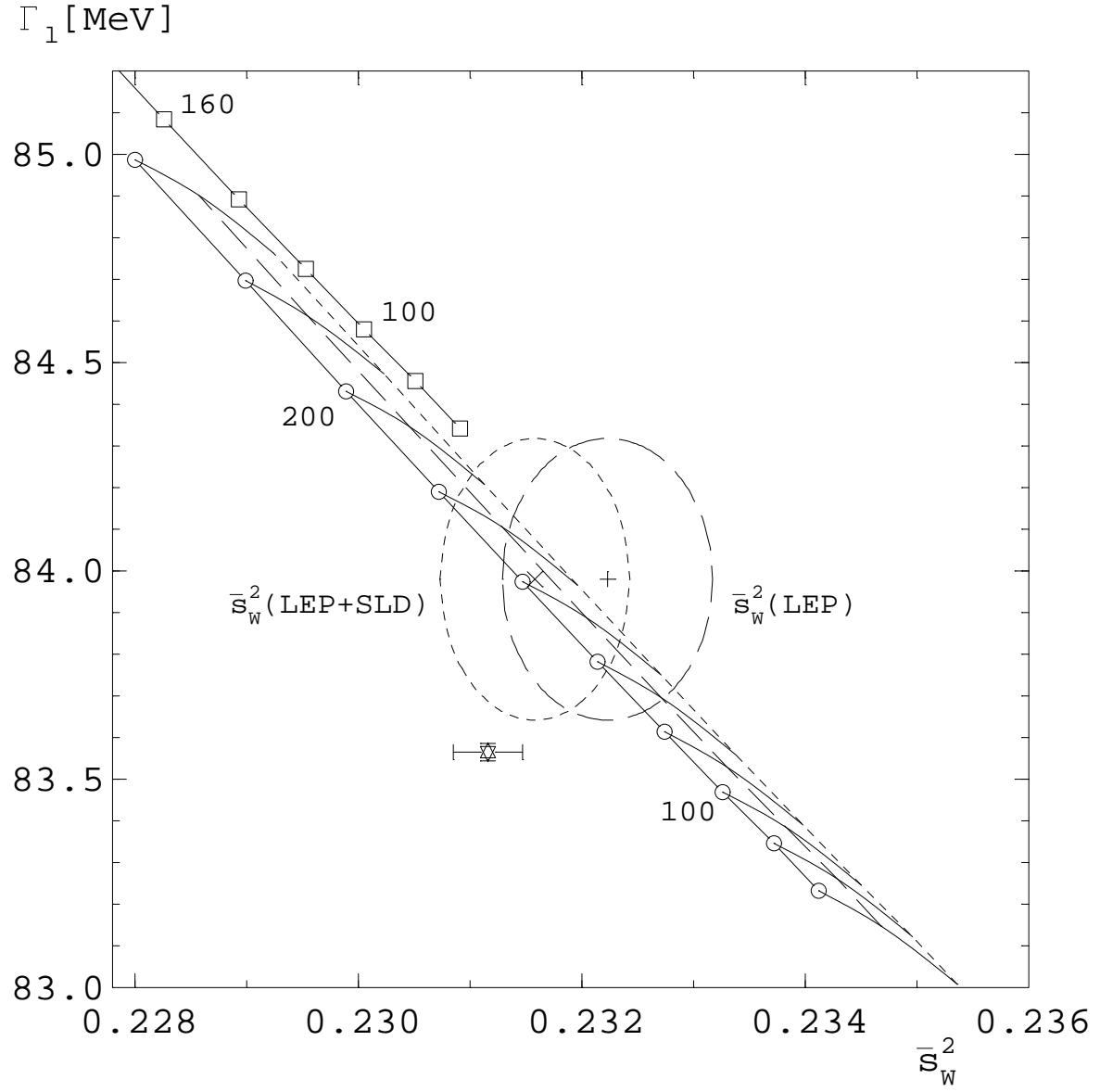


Figure 2: Same signature as Fig. 1, but for the (\bar{s}_W^2, Γ_l) -plane.

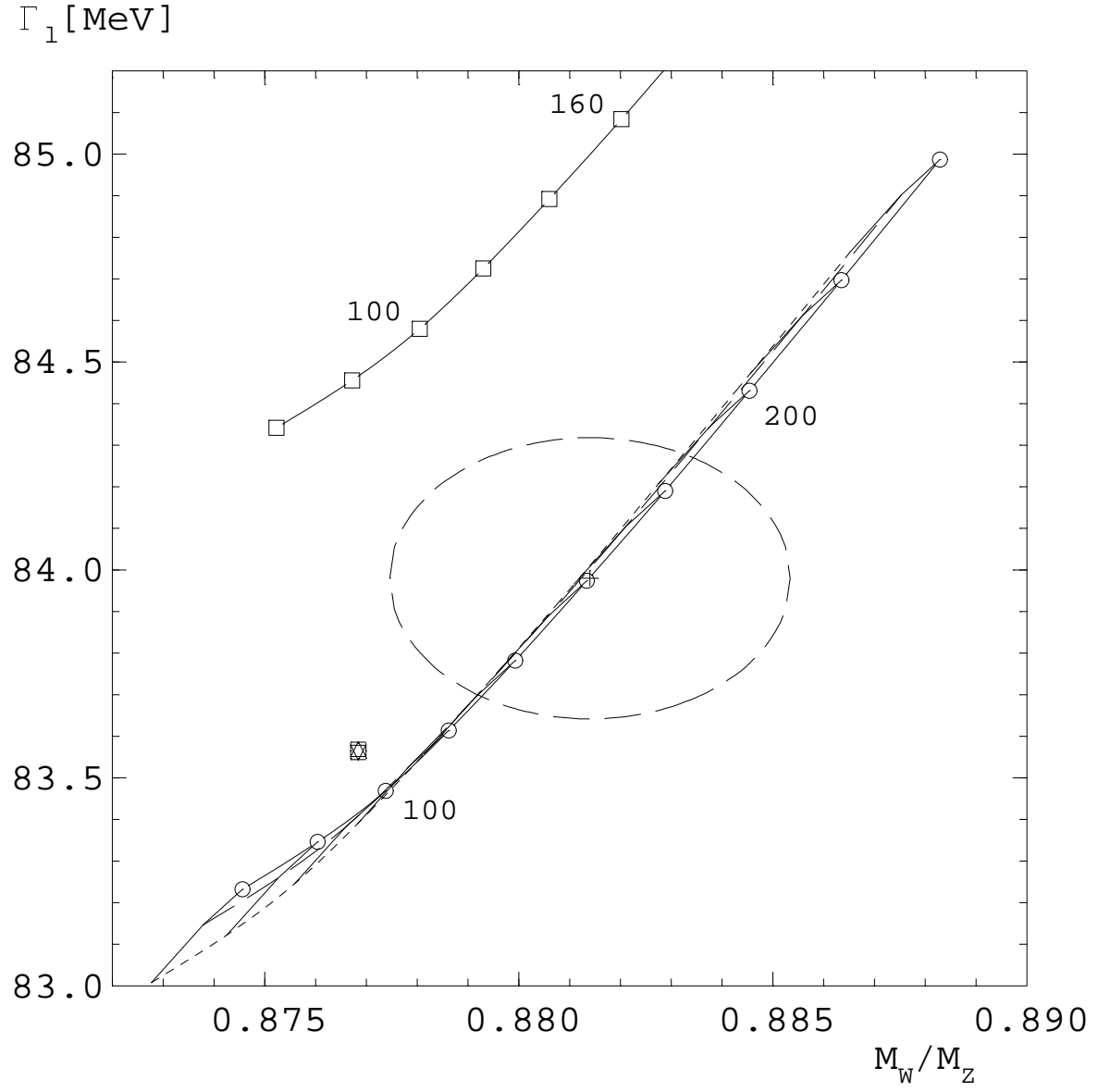


Figure 3: Same signature as Fig. 1, but for the $(M_{W^\pm}/M_Z, \Gamma_l)$ -plane.

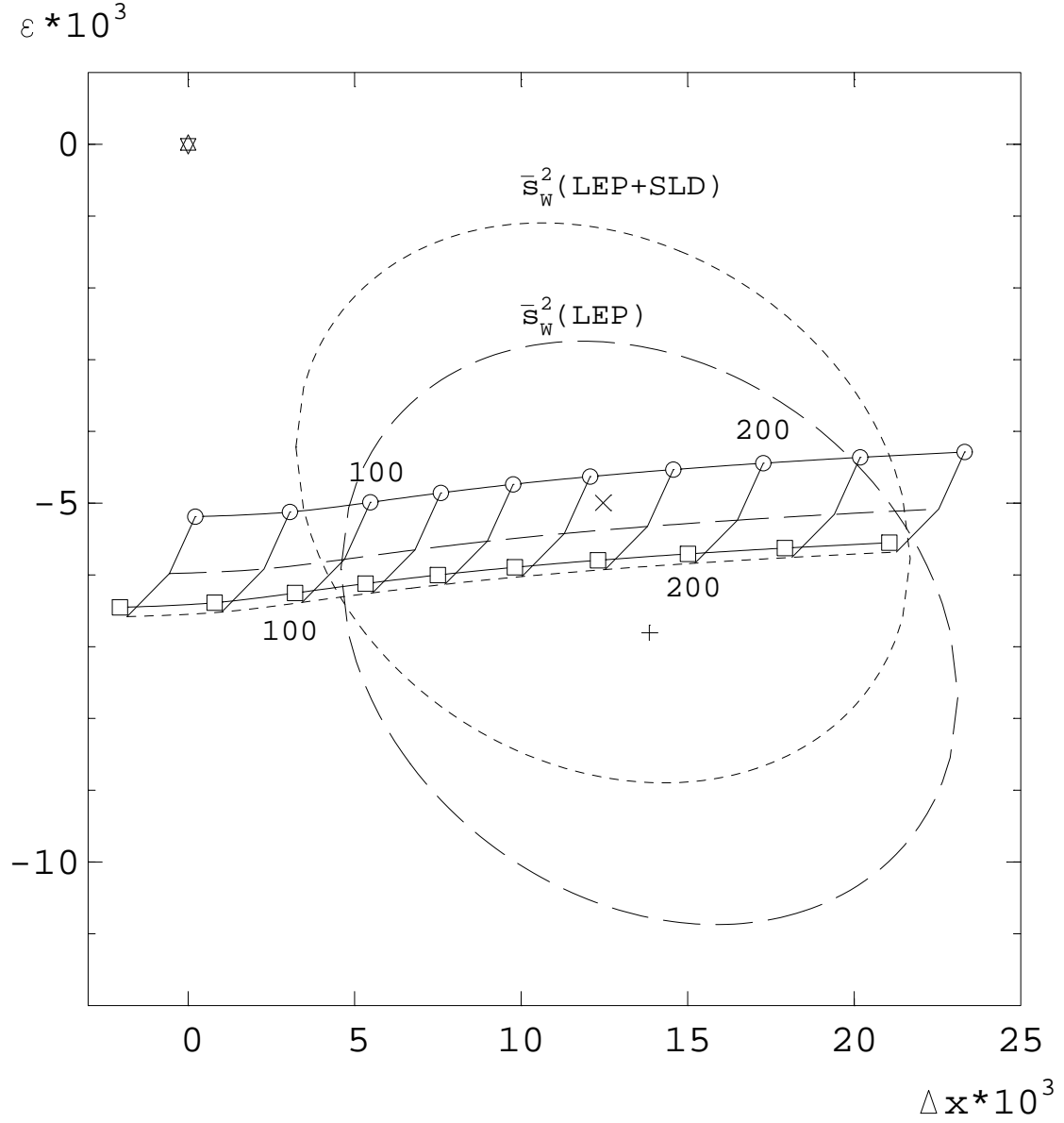


Figure 4: The experimental data are represented by the projection of the 68% C.L. volume in $(\varepsilon, \Delta x, \Delta y)$ -space into the $(\varepsilon, \Delta x)$ -plane. As in Figs. 1-3, the fermion-loop prediction is shown by the single line, the full one-loop standard-model prediction by the connected lines.

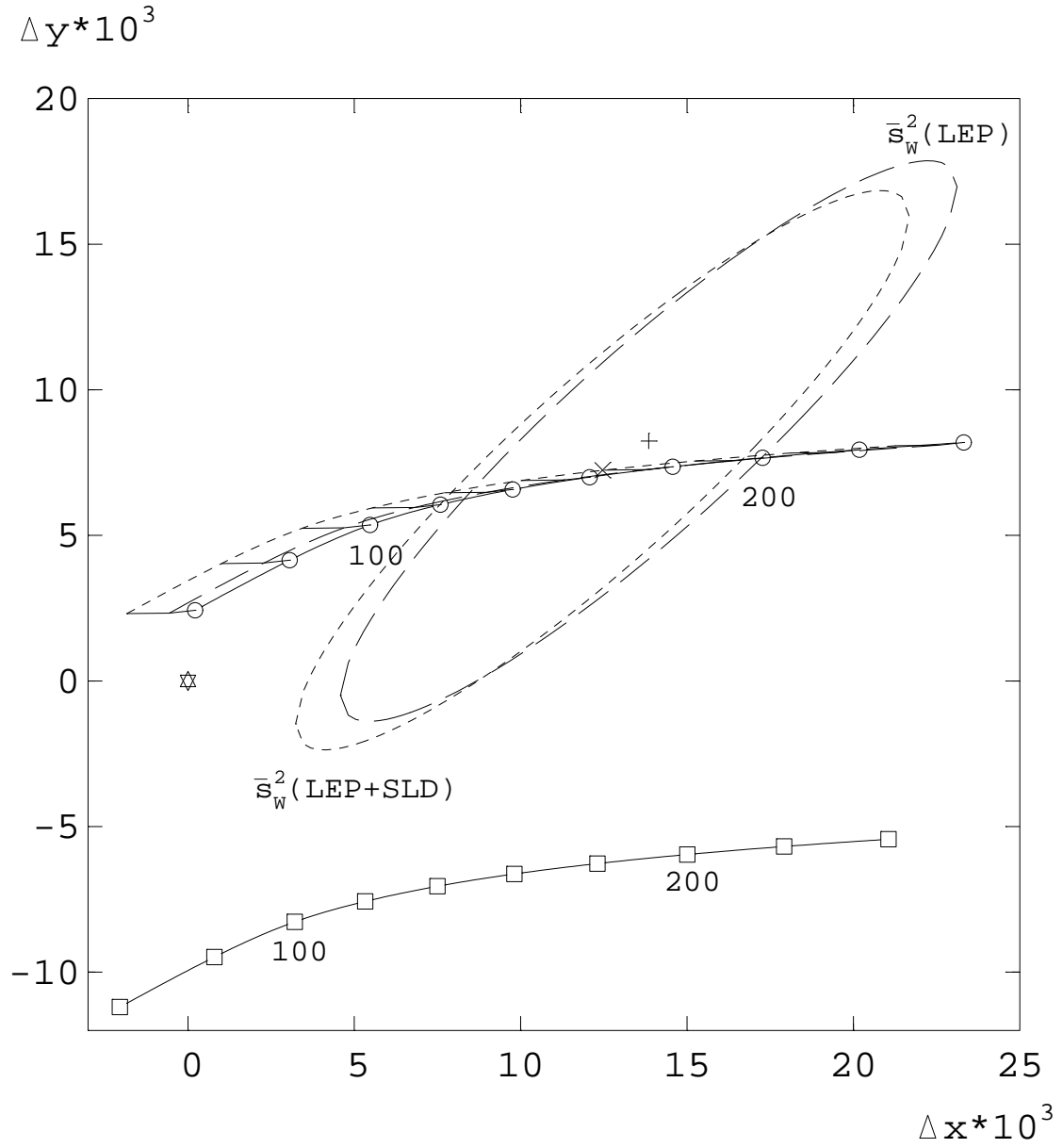


Figure 5: Same signature as Fig. 4, but for the parameters Δx and Δy .

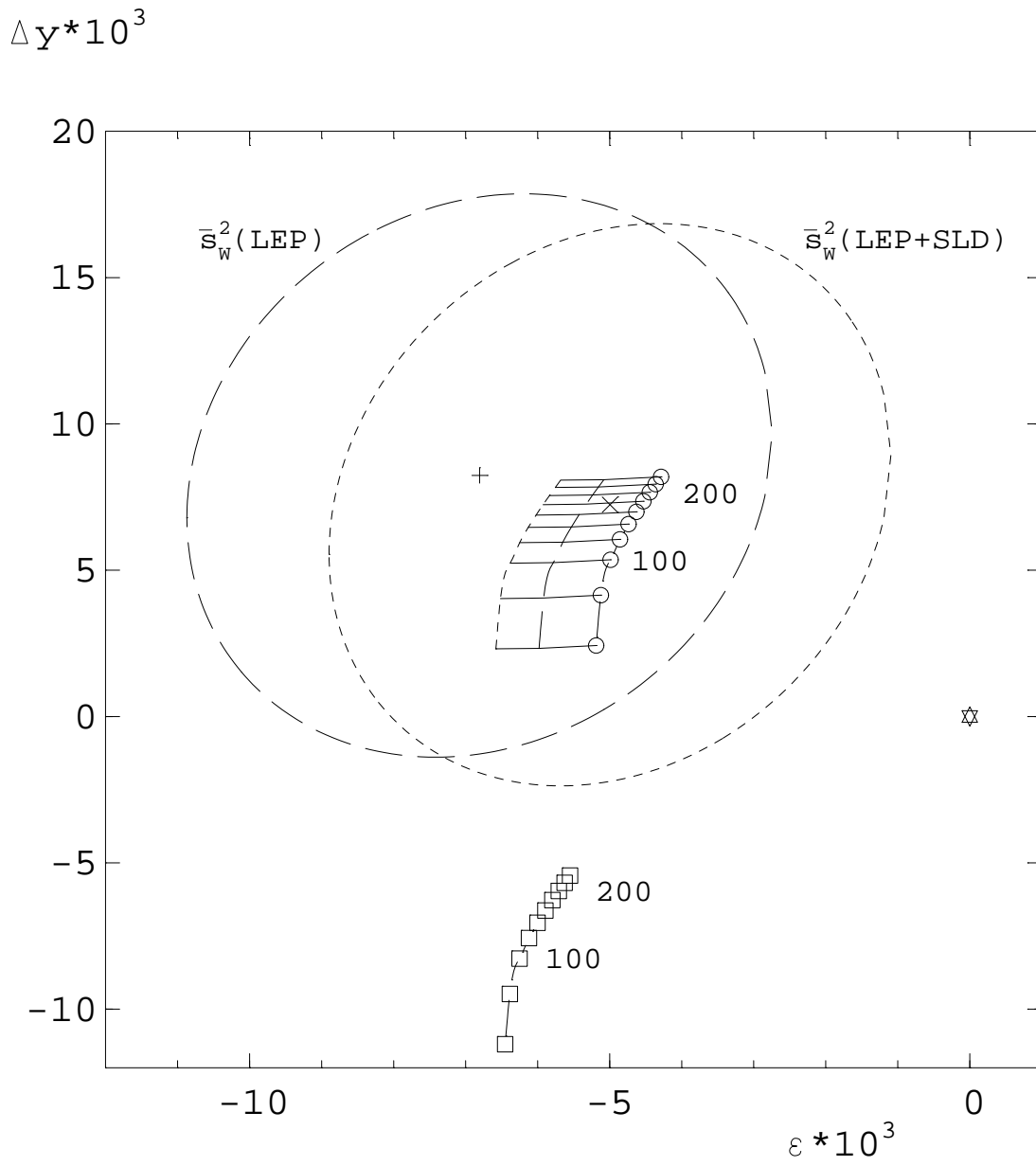


Figure 6: Same signature as Fig. 4, but for the parameters ε and Δy .